* P - 0 0 3 - 0 0 1 6 1 6 *

P-003-001616 Seat No. ________

Third Year B. Sc. (Sem. VI) (CBCS) Examination

March / April - 2020

Mathematics : BSMT - 601 (A)

(Graph Theory & Complex Analysis - II)

(Old Course)

Faculty Code : 003

Subject Code : 001616

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Figures to the right indicate full marks.

1 Answer the following objective type questions briefly in your answer-book : **20**

(1) Define isolated vertex in a graph.

(2) Define : A simple Graph.

(3) Write the degree of a pendant Vertex.

(4) The number of vertices in a binary tree is always __________.

(5) The vertex connectivity of a tree is __________.

(6) Write the number of vertices in a connected graph with 2 faces and 6 edges.

(7) The chromatic number of a complete graph K_n is __________.

(8) A simple connected graph with 6 vertices can have at most __________ edges.

(9) Define : Planer Graph.

(10) Give an example of a self dual graph which is complete.

(11) Define : Bilinear Mapping.

(12) Find the invariant point of $w = \dfrac{6z - 9}{2}$

(13) Define : Conformal Mapping.

(14) Write the Maclaurin series of sinh$z$.

(15) Find the radius of convergence of the series $\sum \frac{z^n}{n!}$.

(16) Identify the function whose power series is $\sum_{n=0}^{\infty} (-1) \frac{z^{2n}}{(2n)!}$.

(17) What is the isolated singular point(s) of $f(z) = \frac{z+1}{z^2+1}$

(18) Res $\left(z \cos\left(\frac{1}{z}\right), 0 \right) =$ __________

(19) State Cauchy's Residue Theorem.

(20) What is the residue of tan$z$ at its poles ?

2 (A) Attempt Any **Three** : **6**

(1) State and prove the first theorem of graph theory.

(2) Show that the complete graph K_n has $\frac{n(n-1)}{2}$ edges.

(3) Explain Konigsberg bridge problem.

(4) Show that the number of vertices in a binary tree is always odd.

(5) In any simple connected planar graph with n vertices, e edges and f faces, show that $e \le 3n - 6$.

(6) Define path matrix and write the properties of path matrix.

(B) Attempt Any **Three** : **9**

(1) Show that the number of vertices of odd degree in a graph is always even.

(2) What is the smallest interger n such that the complete graph K_n has at least 500 edges?

(3) Prove the statement : "In a complete graph K_n with n vertices, there are $\frac{n-1}{2}$ edge disjoint Himiltonian circuits, if n is an odd number greater than or equal to 3."

(4) Show that the number of internal vertices in a binary tree is one less than the number of pendant vertices.

(5) Prove every tree with two or more vertices is 2 – chromatic.

(6) Prove that a connected graph is Euler graph iff it can be decomposed into edge disjoint circuits.

(C) Attempt Any **Two** : **10**

(1) State and prove characterization of a disconnected graph.

(2) Prove that a tree with n vertices has $n-1$ edges.

(3) Prove that a connected graph G is Euler if and only if all the vertices of G are of even degree.

(4) Prove that a graph with at least one edge is 2 – chromatic if and only if it has no odd circuits.

(5) State and prove Euler's formula for planer graph..

3 (A) Attempt Any **Three** : **6**

(1) Show that for a Mobius mapping there are at most two fixed points.

(2) Find the critical point of $\omega = \frac{z-1}{z+1}$.

(3) Find the region of convergence of the power series $\sum \frac{(n+1)z^n}{(n+2)(n+3)}$.

(4) Show that $e^z = e + e\sum_{n=1}^{\infty} \frac{(z-1)^n}{n!}$.

(5) Identify the type of singularity of $\frac{\sinh z}{z^4}; 0 < |z| < \infty$ at $z = 0$.

(6) Find the residue of $\frac{2z+1}{(z-1)(z+1)}$ at $z = 1$.

(B) Attempt Any **Three** : **9**

(1) Show that the composition of two bilinear mapping is again a bilinear mapping.

(2) Obtain the bilinear mapping which maps the points $z = 1, \infty, -1$ of $z-$plane into the points $w = 3, 0, -3$ of w plane.

(3) Expand $\frac{z}{(z-1)(z-3)}$ in the powers of $(z-1)$ if $0 < |z-1| < 2$.

(4) Evaluate $\int_{|z|=2} \frac{3z^2+2}{(z-1)(z^2+9)} dz$.

(5) Find using Cauchy's Residue Theorem $\int_C \frac{1-2z}{z(z-1)(z-2)} dz;\ C : |z| = \frac{3}{2}$

(6) Find the Laurent's series of $f(z) = \frac{-1}{(z-1)(z-2)}$ in the domains

(i) $1 < |z| < 2$

(ii) $2 < |z| < \infty$

(C) Attempt Any **Two** : **10**

(1) Prove that every Mobius mapping maps circles or straight line into circles or straight line.

(2) Prove that the transformation $\omega = 2z + z^2$ maps the unit circle $|z| = 1$ of z-plane into a cardioid in $\omega -$plane.

(3) State and prove Taylor's series expansion of an analytic function.

(4) Prove that :

$$\sinh\left(z + z^{-1}\right) = \sum_{-\infty}^{\infty} a_n z^n \text{ where } a_n = \frac{1}{2\pi} \int_0^{2\pi} \sinh(2\cos\theta)\cos n\theta \, d\theta.$$

(5) Prove that $\int_0^\infty \frac{\cos ax}{x^2+1} dx = \frac{\pi}{2} e^{-a}$.